Tangent to a Curve

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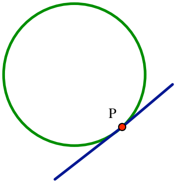

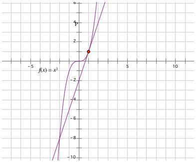

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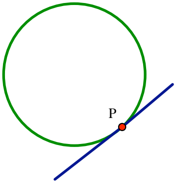

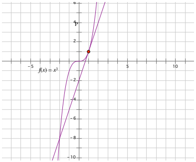

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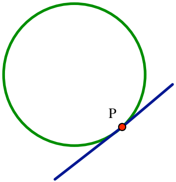

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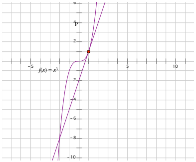

- The above definition does not work for the line that our intuition nominates for the tangent to $y = x^3$ at the point $P(1, 1)$, since this line cuts the graph twice.

Definition of Tangent Line

Our immediate goal in the course is to **make precise the definition of the direction or slope of a curve** (graph of a function) at a point P (if this is possible). In so doing, we can make a precise definition of a tangent to a curve at a point P as the unique line through the point P with the same slope as the curve (when that slope exists).

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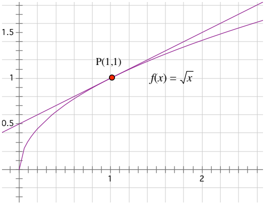
- In the process of defining the slope of a function at a point, we will encounter the concept of a **limit** and the concept of **continuity**. Both are intuitive concepts which we will make precise so that we can determine exactly where and how they apply.
- Although the process of defining the slope and learning to calculate slopes (derivatives) for a wide range of functions will take some time, we can see the concept in action immediately with some particular examples.

Example 1

Example 1 Find the equation of the tangent line to the curve $y = \sqrt{x}$ at the point where $x = 1$ (at the point $P(1, 1)$). This means, we need to find the slope of the tangent line touching the curve drawn in the picture.



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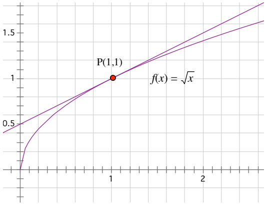

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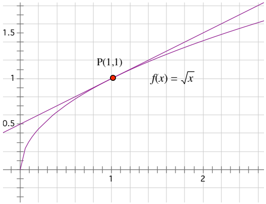

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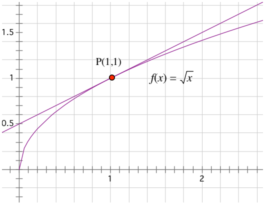

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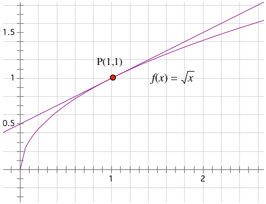

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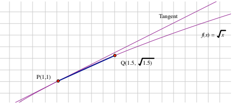
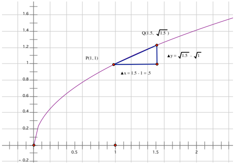
- In fact we just do not have enough information to find this slope. So should we quit?
- We can make an estimate of this slope. How?
- We can approximate the slope of this tangent line using the slope of a line segment joining $P(1,1)$ to a point Q on the curve near P.

Slope of a Secant m_{PQ}.

Let us consider the point $Q(1.5, \sqrt{1.5})$, which is on the graph of the function $f(x) = \sqrt{x}$.





- Since Q is on the curve $y = \sqrt{x}$, the slope of the line segment joining the points P and Q (secant), m_{PQ} the change in elevation on the curve $y = \sqrt{x}$ between the points P and Q divided by the change in the value of x, $\frac{\Delta y}{\Delta x}$ (see diagram on right).

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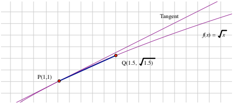

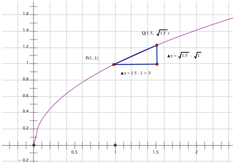

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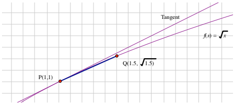

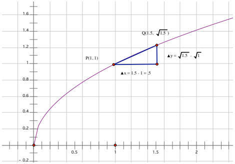

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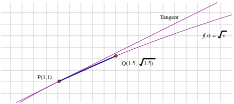

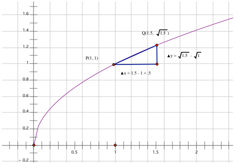

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- If we think of the curve $y = \sqrt{x}$ as a hill and imagine we are walking up the hill from left to right, m_{PQ} agrees with our intuitive idea of the average slope or incline on the hill between the points P and Q.
- Because, Q is so close to P, and because the curve $y = \sqrt{x}$ stays close to the tangent near P, slope of tangent at the point $\quad P \approx m_{PQ} \approx 0.4495$

Slopes of Many Secants

If we choose a different point Q on the curve $y = \sqrt{x}$ we get a different estimate for the slope of the tangent line to the curve at P. Complete the following table of estimates.

	slope of secant$(Q = Q(x, \sqrt{x}))$	Δx	Δy
x	$m_{PQ} = \frac{\sqrt{x}-\sqrt{1}}{x-1} = \frac{\text{Change in y (from P to Q)}}{\text{Change in x (from P to Q)}}$	$x - 1$	$\sqrt{x} - \sqrt{1}$
3.5	$\frac{\sqrt{3.5}-1}{2.5} = .348$	2.5	.8708
3.0	$\frac{\sqrt{3}-1}{2} = .366$	2	.7320
2.5	$\frac{\sqrt{2.5}-1}{1.5} = .387$	1.5	.5811
2.0	$\frac{\sqrt{2}-1}{1} = .414$	1	.414
1.5	$\frac{\sqrt{1.5}-1}{.5} = .449$	.5	.2247
1.2	.4772	.2	.0954
1.1	.4881	.1	.0488
1.01	.4987	.01	.00498
1.001	$\frac{\sqrt{1.001}-1}{.001} =$	.001	4.99×10^{-4}
1.0001		.0001	
1.00001		.00001	

Limit of Slopes Secants as Q approaches P

complete the following sentence:
As x approaches 1, the values of m_{PQ} approach ________

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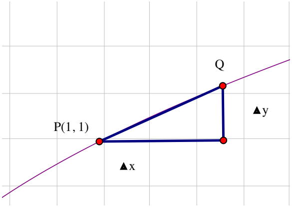
1.0001	$\frac{\sqrt{1.0001}-1}{.0001} = .499987$	.0001	0.0000499988
1.00001	.499999987	.00001	$4.99999 * 10^-6$

Notation for Limit of Slopes of Secants

As x approaches 1, the values of m_{PQ} approach 0.5



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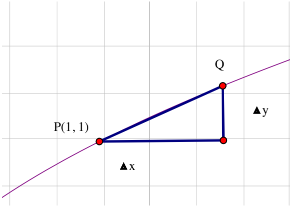

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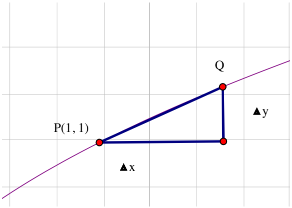

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- As Δx approaches 0, the values of m_{PQ} approach 1/2.

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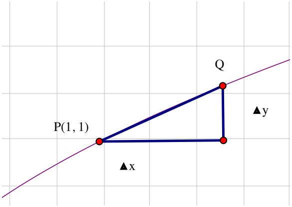

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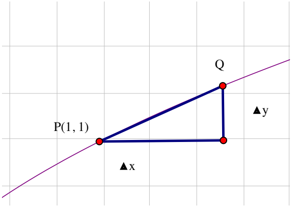

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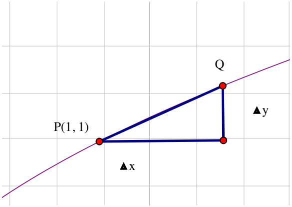

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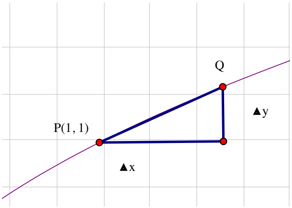

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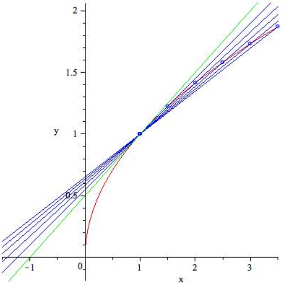


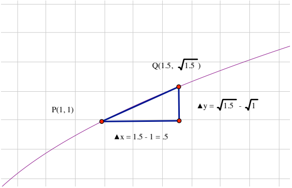
The slopes of the line segments PQ approach the slope of the tangent we seek, as Q approaches P. Hence it is reasonable to define the slope of the tangent to be this limit of the slopes of the line segments PQ as Q approaches P.
Hence the slope of the tangent to the curve $y = \sqrt{x}$ at the point $P(1,1)$ is $1/2$ and the equation of the tangent to the curve $y = \sqrt{x}$ at this point is

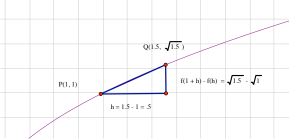
Equation of the tangent at P is $\boxed{y - 1 = \frac{1}{2}(x-1)}$ or $y = \frac{1}{2}x + \frac{1}{2}$.

More Notation

We will also make heavy use of the following notation: We use h to denote the small change in the value of x (between P and Q) Instead of using Δx. This translates to

$$m_{PQ} = \frac{\sqrt{1+h} - \sqrt{1}}{h} = \frac{\sqrt{1.5} - \sqrt{1}}{.5}$$





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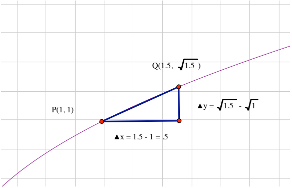

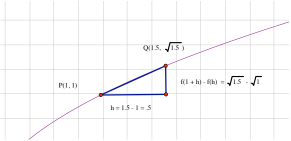

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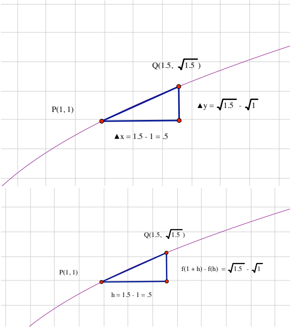

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- or in the language of limits :
  $\lim_{h\to 0} m_{PQ} = \lim_{h\to 0} \frac{\sqrt{1+h}-\sqrt{1}}{h} = 1/2$

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Instantaneous rate of Change

The slope of the tangent to a curve at a point gives us a measure of the **instantaneous rate of change** of the curve at that point. This measure is not new to us, in a car, the odometer tells us the distance the car has travelled (under its own steam) since it rolled off the assembly line. This a function D of time, t. The speedometer on a car gives us the instantaneous rate of change of the function $D(t)$, with respect to time, t, at any given time. When you are driving a car, you see that the **speed** of the car is usually changing from moment to moment. This reflects the fact that the instantaneous rate of change of $D(t)$ or slope of the tangent to the curve $y = D(t)$ varies from moment to moment.

Increasing/Decreasing Functions

When a function is increasing, we get a **positive slope** for the tangent and when a function is decreasing, we get a **negative slope** for the tangent. $D(t)$ above never decreases, reflecting the fact that the speedometer always reads 0 or something positive.

Example

Example A Buzz Lightyear toy is dropped (no initial velocity) from the top of the Willis Tower in Chicago, which is 442 m tall.
We will denote **the distance fallen by the toy after t seconds by $s(t)$ meters** . We have a formula for $s(t)$:

$$s(t) = 4.9t^2 \text{ meters.}$$

See your notes for more details. The velocity or speed of the toy at any given time is the instantaneous rate of change of the function $s(t)$ at that time.

Example

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Time Interval	Average velocity $= \frac{\Delta s}{\Delta t}$ (measured in m/s)
$3 \le t \le 4$	
$3 \le t \le 3.1$	
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$3 \leq t \leq 3.001$	$\frac{s(3.001)-s(3)}{3.001-3} = \frac{4.9(3.001)^2-4.9(9)}{0.001} \approx 29.405\ m/s$
$3 \leq t \leq 3.0001$	$\frac{s(3.0001)-s(3)}{3.0001-3} = \frac{4.9(3.0001)^2-4.9(9)}{0.0001} \approx 29.4005\ m/s$

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$3 \leq t \leq 3.001$	$\frac{s(3.001)-s(3)}{3.001-3} = \frac{4.9(3.001)^2-4.9(9)}{0.001} \approx 29.405\ m/s$
$3 \leq t \leq 3.0001$	$\frac{s(3.0001)-s(3)}{3.0001-3} = \frac{4.9(3.0001)^2-4.9(9)}{0.0001} \approx 29.4005\ m/s$

- In fact the velocity after 3 seconds is 29.4 m/s. We will be able to calculate this precisely after a week or two.